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An Improved Power flow Technique for Distribution Systems

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Abstract

This paper presents an approach of power flow with a view to obtain a reliable convergence in distribution systems. The trigonometric terms are eliminated in the node power expressions and thereby the resulting equations are partially linearised for obtaining better convergence. The proposed method is simpler than existing approaches and solved iteratively similar to Newton-Raphson (NR) technique. This method is applied to two test systems to illustrate its performance.

Key words

power flow, distribution systems

Nomenclature

BNPF	Branch-to-Node matrix based Power Flow
FDPF	Fast Decoupled Power Flow
FDGPF	Fast Decoupled G-matrix method for Power Flow
FDDPF	Fast Decoupled Distribution Power Flow
$a,\ b$	Vector of adopted variables to replace δ
C_i	Constraint equation combining a_i and b_i
$G_{ij} + jB_{ij}$	Real and imaginary parts of bus admittance matrix corresponding to i-th row and j-th column
GS	Gauss-Seidel
J	Jacobian matrix
ε	Small tolerance value for convergence check
NR	Newton-Raphson
nc	Not converged in 50 iterations
nn	Number of nodes in the system
PM	Proposed Method
$P_i + jQ_i$	Real and reactive node power at node-i
V_i and δ_i	Voltage magnitude and voltage angle at node-i respectively
δ_{ij}	$\delta_i - \delta_j$
$\Delta a, \Delta b$ and ΔV	Vector of corrections of a, b and V respectively
ΔC	Vector of mismatches of C
ΔP and ΔQ	Vector of real and reactive node power mismatches respectively

Introduction

Power flow analysis is the determination of steady state conditions of a power system for a set of specified power generations and load demand. It involves the solution of a set of non-linear power flow equations. Applications, especially in the fields of power system optimization and distribution automation, require repeated fast power flow solutions. Due to the large number of interconnections and continuously increasing demand, the size and complexity of the present day power systems, have grown tremendously. In the last few decades efficient and reliable power flow techniques such as Gauss Seidel (GS), Newton-Raphson (NR) [1] and fast decoupled power flow (FDPF) [2] have been developed and widely used for powers system operation, control and planning. However, it has repeatedly been shown that these methods may become inefficient in the analysis of distribution systems due to the following facts.

- Distribution networks can be numerically ill-conditioned due to wide range of r/x ratios and the inherent radial structure
- Distribution power flow equations are different in nature from transmission power flow equations

Consequently many power flow algorithms specially suited for distribution systems have been developed and well documented [1-16]. These methods may be roughly categorized as node and branch based methods. The first category uses node voltages or current injections as state variables and requires information on the derivatives of network equations. The Z-bus method [3], NR based algorithms [4, 5] and FDPF based algorithms [6-8] belong to this category. The second category adopts branch currents or branch powers as state variables and involves only basic circuit laws. The backward/forward sweep based methods [9-15] and loop impedance [16] based methods fall in this category.

A fast decoupled G-matrix method for power flow (FDGPF) of distribution systems, based on equivalent current injections has been proposed in [7]. This method uses a constant and symmetric Jacobian matrix, which needs to be factorized only once. However, the Jacobian matrix has been formed by omitting the reactance of the distribution lines with the assumption that $r >> x$. A fast decoupled distribution power flow (FDDPF) based on equivalent line current flows, which are rotated by appropriate line admittance angle for decoupling the problem, has been suggested in [8]. A compensation based technique that exploits radial structure to achieve high speed, robust convergence and low memory requirement, for weakly meshed distribution systems has been explained in [9]. A simple and efficient branch-to-node matrix based power flow (BNPF) for radial distribution systems has been presented in [10].

Most of the existing algorithms, suggested in the literature, have been developed with an objective to reduce the computational burden by reducing the number of equations, approximating the Jacobian matrix, etc. However, this paper presents a new distribution power flow technique with a different objective in the sense it attempts to enhance the convergence rate by partially linearising the power flow equations. The proposed method is applied on two test systems to reveal its performance and the results are presented.

Proposed Power Flow Method

The power flow equations are partially linearised by eliminating the trigonometric terms. This elimination will improve the convergence of the power flow algorithm and makes the algorithm suitable for ill-conditioned distribution systems.

The expressions for the real and reactive bus powers, P_i and Q_i are,

$$P_i = V_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_i V_j \left[G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij} \right] \tag{1}$$

$$Q_i = -V_i^2 B_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_i V_j \left[G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij} \right] \tag{2}$$

The above two equations can be modified as

$$P_i = V_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_i V_j \left[G_{ij} \left(a_i a_j + b_i b_j \right) + B_{ij} \left(b_i a_j - a_i b_j \right) \right] \tag{3}$$

$$Q_i = -V_i^2 B_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_i V_j \left[G_{ij} \left(b_i a_j - a_i b_j \right) - B_{ij} \left(a_i a_j + b_i b_j \right) \right] \tag{4}$$

where

$$\begin{aligned} a_i &= \cos\delta_i \\ b_i &= \sin\delta_i \end{aligned} \tag{5}$$

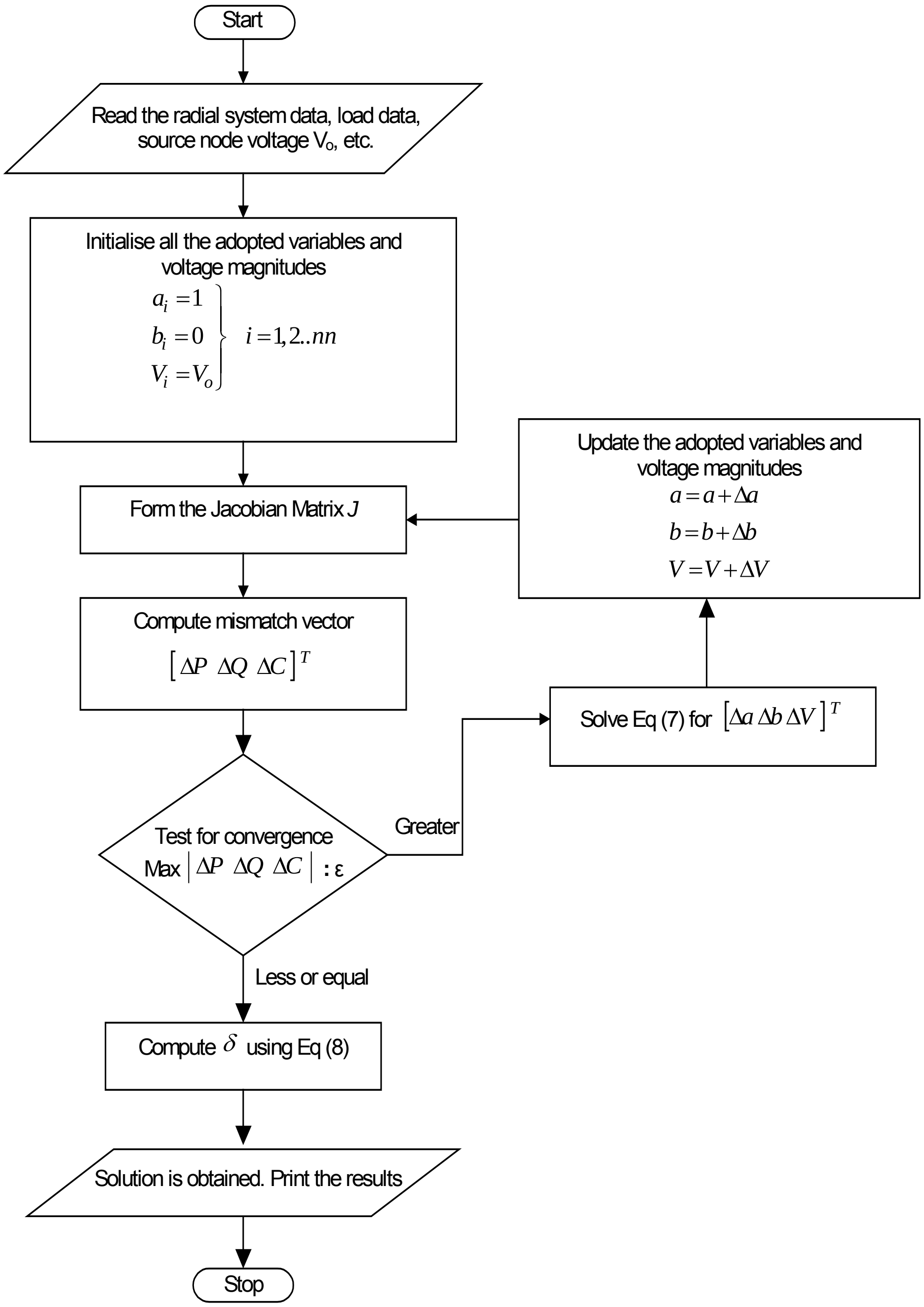

Fig. 1 Flowchart of the proposed method

The a_i and b_i terms in the above equations may be combined through a constraint as

$$C_i = a_i^2 + b_i^2 - 1 = 0 \tag{6}$$

Eqs. 3, 4 and 6 for all the nodes except source node are linearised around a known operating point of a^o, b^o and V^o,

$$\begin{bmatrix} & & \\ & J & \\ & & \end{bmatrix} \begin{bmatrix} \Delta a \\ \Delta b \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta C \end{bmatrix} \tag{7}$$

where $J = \begin{bmatrix} \frac{\partial P}{\partial a} & \frac{\partial P}{\partial b} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial a} & \frac{\partial Q}{\partial b} & \frac{\partial Q}{\partial V} \\ \frac{\partial C}{\partial a} & \frac{\partial C}{\partial b} & \frac{\partial C}{\partial V} \end{bmatrix}$ = Jacobian matrix

The expression for the derivatives of the Jacobian matrix, J, are given in Appendix-1. Eq. (7) may be solved iteratively for a, b and V. Once the adopted unknowns a and b are computed, the node angles can be computed as

$$\delta_i = \sin^{-1} b_i \tag{8}$$

The flow of the proposed algorithm is given in Fig. 1.

Simulation Results

The proposed algorithm is tested to evaluate its solution accuracy and computational efficiency on 15 and 69 node distribution systems [14, 17] using a Pentium-IV, 2 GHz, personal computer. The convergence sensitivity of the proposed method (PM) to r/x ratio of the distribution lines is also tested. Two series of tests are generated, one by varying the base case resistance and the other by changing the base case reactance using a uniform scaling factor, keeping the latter parameter unchanged. The results obtained by the PM are compared with that of a node based method FDDPF [8], a branch based method BNPF [10] and a decoupled method FDGPF [7] to highlight its superior performance. The first two methods are chosen to demonstrate the convergence performance and the third one is selected to show the robustness of the PM. The algorithms are tested with a flat start and a convergence tolerance of 0.0001 per unit.

The solution of the PM for the 15-node system with base case reactance multiplied by a scaling factor of 0.5 is compared with the solution obtained by FDDPF, BNPF and FDGPF methods in Table-1. This table indicates that the PM offers the same solution as that obtained by the other methods, which validates its solution accuracy. Table-2 explains the convergence characteristics in terms of number of iterations. The PM reliably converges for all test systems with a wide variation in r/x ratio of the distribution lines similar to FDDPF and BNPF methods. But the FDGPF needs a higher r/x ratio for convergence even for smaller systems and diverges for 69 node system irrespective of the r/x ratio. It is very clear that the PM is insensitive to r/x ratio and offers solution for even larger systems unlike FDGPF method. These results summarize that the PM is accurate, fast and robust and is suitable for larger distribution systems.

Table 1 Power flow solution obtained for 15-node system

Node No	PM		FDDPF		BNPF		FDGPF	
	V	δ	V	δ	V	δ	V	δ
1	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
2	0.9786	0.0074	0.9787	0.0074	0.9786	0.0074	0.9787	0.0074
3	0.9678	0.0113	0.9679	0.0113	0.9678	0.0113	0.9679	0.0113
4	0.9635	0.0129	0.9636	0.0129	0.9635	0.0129	0.9637	0.0128
5	0.9628	0.0133	0.9628	0.0133	0.9628	0.0133	0.9629	0.0132
6	0.9616	0.0139	0.9617	0.0139	0.9616	0.0139	0.9617	0.0139
7	0.9617	0.0138	0.9618	0.0138	0.9617	0.0138	0.9619	0.0138
8	0.9625	0.0141	0.9626	0.0141	0.9625	0.0141	0.9626	0.0140
9	0.9593	0.0158	0.9594	0.0158	0.9593	0.0158	0.9594	0.0157
10	0.9583	0.0163	0.9583	0.0163	0.9583	0.0163	0.9584	0.0163
11	0.9760	0.0088	0.9761	0.0088	0.9760	0.0088	0.9761	0.0088
12	0.9752	0.0092	0.9752	0.0092	0.9752	0.0092	0.9752	0.0092
13	0.9684	0.0128	0.9684	0.0127	0.9684	0.0128	0.9684	0.0127
14	0.9666	0.0137	0.9666	0.0137	0.9666	0.0137	0.9667	0.0136
15	0.9673	0.0133	0.9674	0.0133	0.9673	0.0133	0.9674	0.0133

Table 2 Number of iterations

	15 node				69 node			
	PM	FDDPF	BNPF	FDGPF	PM	FDDPF	BNPF	FDGPF
$r + j\ x$	4	4	4	*nc*	6	5	5	*nc*
$0.5\ r + j\ x$	4	3	3	*nc*	5	4	4	*nc*
$1.5\ r + j\ x$	4	4	4	16	6	6	6	*nc*
$r + j\,0.5\ x$	4	3	3	9	7	5	5	*nc*
$r + j\,1.5\ x$	4	4	4	*nc*	5	5	5	*nc*

Conclusion

A reliable power flow algorithm has been developed for distribution systems. The convergence characteristics of this formulation has been improved by replacing the trigonometric terms $\cos\delta$ and $\sin\delta$ of the traditional power flow equations by a new set of variables a and b respectively along with the necessary additional constraint equations. The results have been compared with the existing techniques to highlight the superiority of the proposed approach. The approach is well suited for practical implementations.

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Appendix-1

Expression for derivatives of the Jacobian matrix

$$\frac{\partial P_i}{\partial a_i} = \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_i V_j \left[G_{ij} a_j - B_{ij} b_j \right]$$

$$\frac{\partial P_i}{\partial a_j} = V_i V_j \left[G_{ij} a_i + B_{ij} b_i \right]$$

$$\frac{\partial P_i}{\partial b_i} = \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_i V_j \left[G_{ij} b_j + B_{ij} a_j \right]$$

$$\frac{\partial P_i}{\partial b_j} = V_i V_j \left[G_{ij} b_i - B_{ij} a_i \right]$$

$$\frac{\partial P_i}{\partial V_i} = 2 V_i G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_j \left[G_{ij} \left(a_i a_j + b_i b_j \right) + B_{ij} \left(b_i a_j - a_i b_j \right) \right]$$

$$\frac{\partial P_i}{\partial V_j} = V_i \left[G_{ij} \left(a_i a_j + b_i b_j \right) + B_{ij} \left(b_i a_j - a_i b_j \right) \right]$$

$$\frac{\partial Q_i}{\partial a_i} = \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_i V_j \left[- G_{ij} b_j - B_{ij} a_j \right]$$

$$\frac{\partial Q_i}{\partial a_j} = V_i V_j \left[G_{ij} b_i - B_{ij} a_i \right]$$

$$\frac{\partial Q_i}{\partial b_i} = \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_i V_j \left[G_{ij} a_j - B_{ij} b_j \right]$$

$$\frac{\partial Q_i}{\partial b_j} = V_i V_j \left[- G_{ij} a_i - B_{ij} b_i \right]$$

$$\frac{\partial Q_i}{\partial V_i} = -2 V_i B_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{nn} V_j \left[G_{ij} \left(b_i a_j - a_i b_j \right) - B_{ij} \left(a_i a_j + b_i b_j \right) \right]$$

$$\frac{\partial Q_i}{\partial V_j} = V_i \left[G_{ij} \left(b_i a_j - a_i b_j \right) - B_{ij} \left(a_i a_j + b_i b_j \right) \right]$$

$$\frac{\partial C_i}{\partial a_i} = 2 a_i \qquad \frac{\partial C_i}{\partial a_j} = 0$$

$$\frac{\partial C_i}{\partial b_i} = 2 b_i \qquad \frac{\partial C_i}{\partial b_j} = 0$$

$$\frac{\partial C_i}{\partial V_i} = 0 \qquad \frac{\partial C_i}{\partial V_j} = 0$$

Appendix-2

Data for 15-node Distribution System

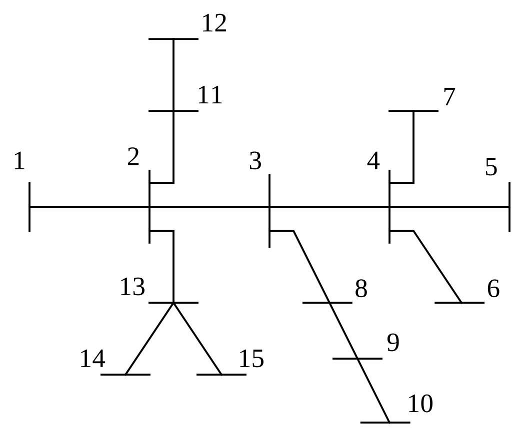

Fig. 2 One line diagram

Table 3 Line data

Sending node	Receiving node	*r* (ohm)	*x* (ohm)
1	2	1.35309	1.32349
2	3	1.17024	1.14464
3	4	0.84111	0.82271
4	5	1.52348	1.02760
4	6	1.19702	0.80740
4	7	2.23081	1.50470
3	8	1.79553	1.21110
8	9	2.44845	1.65150
9	10	2.01317	1.35790
2	11	2.01317	1.35790
11	12	1.68671	1.13770
2	13	2.55727	1.72490
13	14	1.08820	0.73400
13	15	1.25143	0.84410

Table 4 Load data

Node No	Load (kW + j kVAr)
2, 5, 10, 12	44.10 + j 44.99
3, 7, 9, 11, 15	70.00 + j 71.41
4, 6, 8, 13, 14	140 + j 142.82

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