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Estimation And Calculation Of A Relationship Between Dispersion Number, Reynolds Number, Porosity And Hydraulic Gradient in Horizontal Roughing Filter

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Abstract:

The study was aimed to estimate the flow criteria in Horizontal Roughing Filter (HRF) used as a pretreatment unit before Slow Sand Filter (SSF). The Head loss and the nature of flow in porous medium i.e. in gravel bed has been ascertained with overflow rate for the gravel size ranging between 15mm-10mm, 10mm-5mm, 5mm-1mm. The Dispersion number of the flow through gravel bed is in between 0.14 to 0.15 which indicate that the flow through gravel bed in HRF acts as a plug flow type reactor. A relationship among dispersion number, porosity, Reynolds number and hydraulic gradient for flow through gravel bed has been established, for this study only pond water of different turbidity was used without any chemical dosing.

Key Words: Dispersion Number, Hydraulic Gradient, Porosity, Reynold’s Number.

Introduction:

Gravel beds filter has been proposed in Horizontal flow roughing filters as a Pretreatment process so as to reduce solids load on succeeding Slow Sand Filters (SSF) or Rapid Sand Filter (RSF). The flow through clean, granular media has developed little since the work of Fair and Hatch (1933). The present study is to evaluate the updated developments of flow through Porous media and to develop a model study for estimating head loss, dispersion number of flow, turbidity removal efficiency of media. An experimental and theoretical investigation of fluid flow through unconsolidated rigid packing of gravel has established the fact that Mean shear stress at the surface of the gravel is a function of:

(1) The viscosity, density and characteristics velocity of fluid,
(2) A length of characteristics of pores of gravel,
(3) The shape, size distribution and surface roughness of gravels in the bed which are expressed in terms of resistance, co-efficient, the shear stress respectively. poiscuille observed that the flow in a capillary tube was directly proportional to the hydraulic head action on the tube and inversely proportional to its length (poisecuille 1841), Darcy observed that the flow through a bed of a filter sand was directly proportional to the hydraulic head acting on the sand bed and inversely proportional to its depth (Darcy 1856). The following equation Darcy proposed

$$Q = K.A. (H + \Delta L - H_0) / \Delta L$$

Where Q = flow rate through the sand bed,
K = coefficient dependent on the nature of sand (units of velocity), coefficient of permeability,
A = the area of sand bed perpendicular to the flow.
ΔL = depth of the sand bed.
H = the height of surface of the influent water above the top of the sand bed and
H_0 = the height of the surface of the effluent water above the bottom of the sand bed.

Defining the head loss through the sand in the traditional way

$\Delta H = H + \Delta L - H_0$

Darcy's equation is expressed more commonly in the following from:

$$Q / A = K (\Delta H / \Delta L) \qquad (1)$$

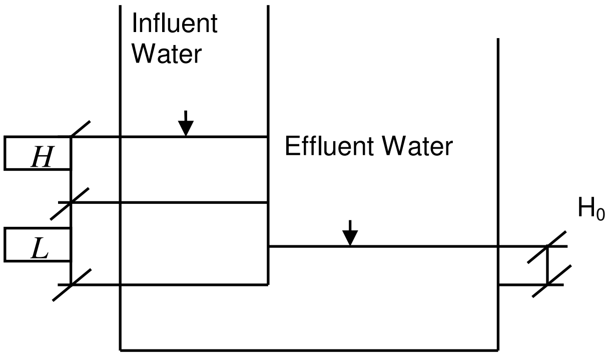


Figure –1

Simplified Sketch of Darcy's Water Filter

Estimation of Head Loss through Gravel Bed

From equation (1) we get, $Q/A = K (\Delta H / \Delta L)$

The value of K may be sated in order to develop a fluid independent and media specific parameter. Nutting (1930) defined the specific permeability as $k = K (\mu / \rho)$, where K = Nutting specific permeability, μ = Fluid viscosity, ρ = Fluid density. the gravel bed of porus media may be classified by number of parallel plates through the depth of bed and poiseculle equation (1841) can be applied to estimate the loss from the velocity of flow,

$$\Delta H / \Delta L = 32 (\mu / \rho g)\ v/d^2 \qquad (2)$$

Where d = diameter of gravel in each compartment.

Dupuit (1863) stated that average velocity in porus medium may be defined as,

$V = v/\varepsilon = Q / A / \varepsilon$

Schllier (1923) defined that the hydraulic radius as the ratio of cross section of flow to the wetted perimeter. Kozeney defined hydraulic radius as the ratio of volume of fluid and solid media surface.

Specific surface (S_v) = Surface of media / volume of media = $S / l_x A (1-\varepsilon)$

So, Hydraulic Radius $R_h = \varepsilon / S_v (1- \varepsilon)$

For circular shape of gravel $R_h = d/4$, so, $d = 4\ \varepsilon / S_v (1- \varepsilon)$

from equation (2) $\Delta H / \Delta L = 32 (\mu / \rho g)\ v/d^2 = 32 (\mu / \rho g) v / (4/S_v)^{2*} [\varepsilon / (1 - \varepsilon)]^2 = 32 (\mu / \rho g)[[(S_v /4)^* [(1-\varepsilon) / \varepsilon]]^2 V$

We know, S_v = Surface of media / volume of media = $\Pi d^2 / \Pi d^3 / 6 = 6/d$

So, $\Delta H / \Delta L = 32 (\mu / \rho g) [(6 /4d) [(1- \varepsilon) / \varepsilon]^2 V$

$$\Delta H / \Delta L = 72 (\mu / \rho g) [(1- \varepsilon) / \varepsilon]^2\ V/d^2 \qquad (3)$$

Estimation of nature of flow in porous media

the plot of Reynolds numbers vs. resistance coefficient for gravel media exhibits a long progression from streamline from streamline Darcy – flow state, in which head loss is proportional to the velocity and in turbulent flow state head loss varies to the square of velocity. Some (Ergum and Orning 1949, Ergun 1952, ward 1964) argue that non linearity is due to increasing degree of turbulence in porous media flow. In fact numbers of authors use this logic and a parallel with known head loss relationships in turbulent pipe line flow to justify a structure for the coefficients of the second term in the head loss equation (Black 1923, Bruke and Plummer 1928, ward 1964).

None-less the deviation from the Darcy's relationship begin to occur at Reynolds number as low as 1 to 10 (Forchheimer 1901, Hubbert 1956) and number of experiments begin while the condition of laminar flow are met, i.e., when a dye is injected in the flow a distinct streamline path is followed. Hubbert (1956) and Schneebeli (1955) also confirmed these result with beds of transparent packing. The first appearance of turbulence has been observed to occur at $40<NRE<140$ in visual studies by Schneebeli (1955) at $NRE>600$ in visual studies by Hubbert (1956) and at 90 C $NRE<120$ IN WORK WITH Biot wire anemometer in air by D.e Wright (1968). According to Scheidegger (1960) Topakoglin (1951) demonstrated from first principles, that the forflow in a coiled tube the Reynolds number at which turbulence begins is a function of Curvature and Reynolds number itself. So, it is obvious that onset of turbulence will begin in Porous media at different Reynolds number depending upon media size, shape and compaction. Generally two types of internal events occur in flow through porous media (1) during entry and exit of flow at the various cells expansion and contraction of each media particles,

(2) As the flow lines travel in and out and around the particle of gravel, so curvilinear flow will occur. This curvilinear effects observed by white (1929) and wallies (1939). Both can be better visualized with the help of the photomicrograph in Fig 2.

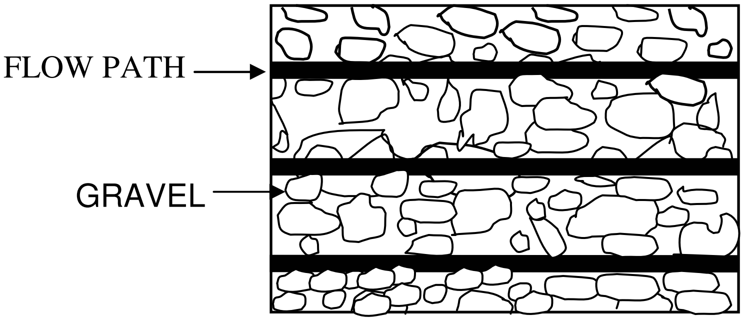
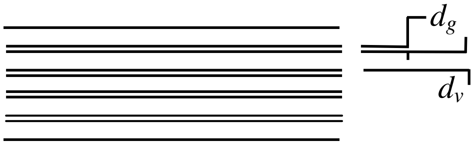


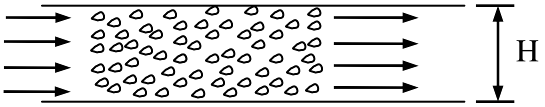
Figure-2

There are four regimes of flow in porous media. The first regime is named as Darcy regime four is limited to Reynolds number below 1.A Reynolds number of 1 corresponds to a filtration rate of approximately 2.5 gpm/sq.ft (6.6 meter / hr) with 0.5mm filter media. Flow in this regime is not only laminar but also "creeping" flow i.e. the laminar flow with no significant inertial contribution. The second regime is named the Forchheimer regime in which initially the flow is steady laminar but as it progresses the initial effects becomes very vital. Initially the head loss is proportional to V with a small V^2 dependence but as it progresses the head loss V^2 becomes related with a small dependence on V. the upper end of this flow regime correspond to a Reynolds number of approximately 100. The third regime represents more or less inertial flow to full turbulence. The upper regime of this is not well established but it is likely to be in between N_{RE} 600 and 800 depending on the media and flow conditions. The fourth and final regime is the regime of full turbulence. In this regime the velocity is randomly fluctuating and flow is turbulent. So, for filtration purpose Reynolds number in excess of 0.5 and less than 50 has to be kept to put them solidly in the Darcy and Furchheimer flow regimes.

Model for Gravel Bed And Experimental Set-Up

Several researchers (e.g. Swason and Williamson 1980, Wegalin et. al. 1987) have established the fact that sedimentation is the main separation process in HRF gravel bed. This could be verified in laboratory by path observation of suspended matter. V_0 represents the particle removal by sedimentation and hence will determine removal ratio in gravel bed. In the modeling approach presented here, the gravel bed configuration has been schematized to derive expression **for the Dispersion number** (D/UL) a Dimensionless Parameter. Na CL mixed water is passed through a series of gravel bed with a constant rate. The concentration of NaCL is measured in the influent as well as passage of the different compartments after passing the compartments of different size gravel bed. Sedimentation is the process of particle removal mechanism; Particles are removed from suspension when they touch the top of gravel grain. G value is assumed to be constant along water flow path due to constant velocity and samew size of gravel, as well as during the run time as head loss increases is very low. The gravel bed is assumed as a multiple plate settler. The set up is as follows:-





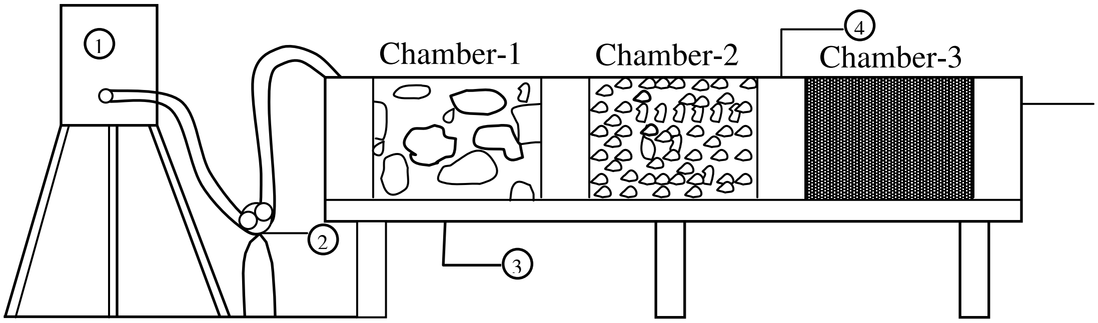


Figure-3 Horizontal Roughing Filter Model

1). Water Reservoir.
2). Peristaltic Pump.
3). Table.
4). Fiber Glass Make Gravel Chambers Containing 15mm.- 10mm. size gravel in 1st chamber & 10mm. - 5mm. 2nd chamber 5mm.-1mm 3rd chamber.

Each compartment containing Gravel is of length *L* width *B* and height *H*. the average gravel size may be assumed dg in parallel layers corresponding plate thickness dg. To develop model settler's configuration the overall dimension is considered identical. Let the nos. of equivalent plates is N_p and *dv* is distance between the plates, discharge is *Q* and velocity is assumed to be same in the gravel bed. As a particle with lower setting velocity than V_0, if V_0 is the setting velocity will be removed. So overflow rates will be incase of plan area of setting tank B.L.

$$V_0 = Q/\ (N_pBL) \qquad (4)$$

The total length *L* of the Setting Tank consists of a number of vertical sections of equal thickness Δx perpendicular to longitudinal axis, and L_x is the distance up to the x th section measured from inlet. The surface area at the top of gravel to a distance L_x will be $S_{0x} = K_0\ B\ L_x\ N_p$

So, in terms of pore velocities the comparable removal efficiency will be obtained by equation time for same length in two systems. So porosity of the gravel bed (ε) specified be for a given discharge, *Q* is given by, $\varepsilon = (d_v\ K_0\ B\ L_x\ N_p)/H\ K_0\ B\ L_x)$

$$\text{Or, } N_p = (H\varepsilon)\ /\ d_v \qquad (5)$$

The number of plates,

$$N_p = H / (d_v + d_g) \qquad (6)$$

Equating (5) and (6), the porosity of gravel bed or $\varepsilon = d_v / (d_v + d_g)$.

Now over flow rate up to a distance L_x will be

$$V_{0x} = Q / S_{0x} = Q / K_0 \, B \, L_x \, N_p$$
$$= (V \, B \, H \, d_g) / (K_0 \, (1- \varepsilon) \, H \, B \, L_x)$$

$$V_{0x} = V \, d_g / (K_0 \, (1- \varepsilon) \, L_x) \qquad (7)$$

Methodology

For finding out Dispersion number (D/UL). A dimensionless parameter an experiment was conducted as per the above schematic diagram. a peristaltic pump was set up to maintain constant velocity through the gravel bed resulting in saline water flow rate of 1.4m^3/m^2/hr. synthetic solution of saline water of 1400 ppm. was prepared by adding Nacl (Sodium Chloride) to top the of water. Under the flow rate it was observed that the concentration in the 1st Chamber is higher and reduces as it flows and due to dilution occurred by flow of water. The discharge is over head tank and it is assumed to be same through gravel bed. At different time the concentration of NaCL mixed water were taken up to ascertain particle size distribution and thus a Dimensionless parameter are obtained. With different influent turbidity of pond water same experimenters were carried out, after that by the help of variance technique the Dispersion number (D/UL) was computed.

By Variance Technique:

Mean time $t = \dfrac{\sum tC}{\sum C}$, Variance is $\sigma t^2 = \dfrac{\sum t^2 C}{\sum C} - t^2$

Variance in terms of dimensionless time = $\left(\sigma^2 = \sigma t^2 / t^2\right)$

Where $\sigma^2 = 2\left(\dfrac{D}{UL}\right) - 2\left(\dfrac{D}{UL}\right)^2 . \left(1 - e^{-1/(D/UL)}\right).$.

INFLUENT TURBIDITY – 80(NTU)

Chamber-1

Time(t)	**Con.(C)**	t^2	**tC**	**t^2C**
0.167	2310	0.027889	385.77	64.42359
0.33	2280	0.1089	752.4	248.292
0.50	2260	0.25	1130	565
0.667	2250	0.444889	1500.75	1001.00025
0.83	2240	0.6889	1859.2	1543.136
1.00	2270	1	2270	2270
∑3.494	∑13610	∑2.520578	∑7898.12	∑5691.85184

mean time(t)	t^2	$\dfrac{\sum t^2C}{\sum C} - t^2$	$\sigma \times t^2$	σ^2
0.58031741	0.3367683	0.418211	0.081442709	0.24183603

***D/UL* = 0.140698**

Chamber-2

Time(*t*)	**Con.(*C*)**	t^2	***tC***	t^2C
0.167	2300	0.027889	384.1	64.1447
0.33	2270	0.1089	749.1	247.203
0.50	2250	0.25	1125	562.5
0.667	2230	0.444889	1487.41	992.1025
0.83	2220	0.6889	1842.6	1529.358
1.00	2260	1	2260	2260
$\sum$3.494	$\sum$13530	$\sum$2.520578	$\sum$7848.21	$\sum$5655.308

Mean time(*t*)	t^2	$\frac{\sum t^2C}{\sum C}-t^2$	$\sigma\times t^2$	σ^2
0.58005987	0.336469	0.417983	0.081513	0.242261

D/UL **= 0.140993**

Chamber-3

Time(*t)*	**Con.(*C*)**	t^2	***tC***	t^2C
0.167	2310	0.027889	385.77	64.42359
0.33	2280	0.1089	752.4	248.292
0.50	2260	0.25	1130	565
0.667	2250	0.444889	1500.75	1001.00025
0.83	2240	0.6889	1859.2	1543.136
1.00	2270	1	2270	2270
$\sum$3.494	$\sum$13610	$\sum$2.520578	$\sum$7898.12	$\sum$5691.85184

mean time(*t*)	t^2	$\frac{\sum t^2C}{\sum C}-t^2$	$\sigma\times t^2$	σ^2
0.58031741	0.3367683	0.418211	0.081442709	0.24183603

D/UL **= 0.140698**

INFLUENT TURBIDITY – 60(NTU)

Chamber-1

Time(t)	**Con.(*C*)**	t^2	***tC***	t^2C
0.167	2290	0.027889	382.43	63.86581
0.33	2360	0.1089	778.8	257.004
0.50	2300	0.25	1150	575
0.667	2340	0.444889	1560.78	1041.04
0.83	2350	0.6889	1950.5	1618.915
1.00	2380	1	2380	2380
1.17	2360	1.3689	2761.2	3230.604
1.33	2330	1.7689	3098.9	4121.537
$\sum$5.994	$\sum$18710	$\sum$5.658378	$\sum$14062.61	$\sum$13287.97

mean time(*t*)	t^2	$\frac{\sum t^2C}{\sum C}-t^2$	$\sigma\times t^2$	σ^2
0.7516093	0.564917	0.710206631	0.145290092	0.257189

D/UL **= 0.1515**

Chamber-2

Time(t)	**Con.(C)**	t^2	tC	t^2C
0.167	2320	0.027889	387.44	64.70248
0.33	2310	0.1089	762.3	251.559
0.5	2310	0.25	1155	577.5
0.667	2337	0.444889	1558.779	1039.706
0.83	2339	0.6889	1941.37	1611.337
1.00	2360	1	2360	2360
1.17	2370	1.3689	2772.9	3244.293
1.33	2330	1.7689	3098.9	4121.537
$\sum$5.994	$\sum$18676	$\sum$5.658378	$\sum$14036.69	$\sum$13270.63

mean time(t)	t^2	$\frac{\sum t^2C}{\sum C}-t^2$	$\sigma\times t^2$	σ^2
0.75158969	0.564887	0.710572	0.145684	0.2579

***D*/*UL*= 0.1523**

Chamber-3

Time(t)	**Con.(C)**	t^2	tC	t^2C
0.167	2340	0.027889	390.78	65.26026
0.33	2290	0.1089	755.7	249.381
0.5	2280	0.25	1140	570
0.667	2310	0.444889	1540.77	1027.694
0.83	2360	0.6889	1958.8	1625.804
1	2340	1	2340	2340
1.17	2340	1.3689	2737.8	3203.226
1.33	2300	1.7689	3059	4068.47
$\sum$5.994	$\sum$18560	$\sum$5.658378	$\sum$13922.85	$\sum$13149.83

mean time(t)	t^2	$\frac{\sum t^2C}{\sum C}-t^2$	$\sigma\times t^2$	σ^2
0.7501536	0.56273	0.708504	0.145774	0.259047

***D*/*UL*= 0.1528**

Hence the dispersion number has been found in the range of 0.14 to 0.15, and hence, it may be stated that the flow through the gravel bed in HRF acts as **PLUG FLOW** type reactor.

Calculation of Reynold's number & Hydraulic gradient:

From Carmen-Kozeny Equation we know that

$$h_f = \frac{f' L(1-e)V_s^2}{e^3 g d_p}$$

Where, h_f = fiction loss through bed of particular of particular of Uniform Size d_p, (m), L = length of the filter, (m), ε = porosity of bed, V_s= filtering velocity (m/s^2), g =

gravitational acceleration (m/s^2), d_p= diameter of filter media grains, (m), f' = friction factor.

Where $f' = 150\frac{(1-\varepsilon)}{\mathrm{Re}} + 1.75$

R_e = Reynold's number

$R_e = \frac{\phi p_w v_s d_p}{\mu}$

ρ_w = density of water (kg/m^3), μ = dynamic viscosity of water (N-s/m^2), ϕ = 0.75 to 0.85, where ϕ = spherocity, i = hydraulic gradient

$i = h_f/L$

By using the above equation, we get, Re = 0.928 (for Chamber-1)
= 0.558 (for Chamber-2)
= 0.186 (for Chamber-3)

Hydraulic Gradient, i = 0.00015 (for Chamber-1)
= 0.00052 (for Chamber-2)
= 0.00349 (for Chamber-3)

The degree of porosity of gravel is considered 34 (average porosity), the range is in between (29-39) % Ref: http://www.rctednet.net/geography/interpretation.hml.

For obtaining user friend equations we take up d_p =12.5mm, 7.5mm, 2.5mm for Chamber-1, Chamber-2 & Chamber-3 respectively.

The result given in the below table:

TABLE-1 Relation between *D/UL*, *Re, e,* and *i*

CHAMBER NUMBER	**DISPERSSION NO. (*D/UL*)**	**REYNOLD'S NO. (*Re*)**	**POROSITY (ε)**	**HYDRAULIC GRADIENT (*i*)**
CHAMBER-1	0.1515	0.928	0.35	0.00015
CHAMBER-2	0.1523	0.556	0.33	0.00052
CHAMBER-3	0.1528	0.186	0.30	0.00049

Table-2 Correlation Matrix among the Hydraulic parameters ($p < 0.05$) for Chamber – 1, 2 and 3 of Horizontal roughing Filter

	Re	ε	*i*	*D/UL*
Re	1.00			
e	0.99	1.00		
i	-0.91	-0.95	1.00	
D/UL	-0.99	-0.95	0.85	1.00

R_e = Reynold's Number; ε = Porosity; i = Hydraulic Gradient; *D/UL* = Dispersion Number

Relation between Reynold's (R_e) Number & Porosity (ε):

$$\varepsilon = A R_e + B \tag{8}$$

Where, a & b are constants.

A = 0.0674 & B = 0.2892

Relation between Porosity (*e*) & Hydraulic gradient (*i*):

$$i = C\varepsilon + D \tag{9}$$

Where, C & D are constants.

C = -0.059 &D = 0.0205.

Relation between Hydraulic gradient (*i*) & Dispersion number (*D/UL*):

$$D/UL = Ei + F \tag{10}$$

where, E & F are constants.

E = 0.3699 & F=0.1519.

Relation between Reynold's (*R*$_e$) & Dispersion number (*D/UL*):

$$D/UL = g\,R_e + h \tag{11}$$

Where, g & h are constants.

g = 0.0013 & h=0.1513

So the user friendly equations are:

$$\varepsilon = 0.0674\,R_e + 0.2892 \tag{12}$$

$$i = -0.059\,\varepsilon + 0.0205 \tag{13}$$

$$D/UL = 0.3699i + 0.1519 \tag{14}$$

$$D/UL = 0.0013\,R_e + 0.1531 \tag{15}$$

Conclusions: In this discussion the development in the model of gravel bed of HRF the head loss in the bed is estimated. Calculations presented here are based on average media size and area volume shape factor. The porosity of the media is also taken into account. Here, in this case it is observed that Reynold's Number obtained for the flow rate of 1.4m^2/m^3/hr, the flow path is laminar with no significant internal contribution to flow. A relationship between Dispersion number, Reynolds Number, Porosity and hydraulic gradient are tried to be established in this discussion. From equation (11) it is seen that the influence of Reynold's Number on Dispersion number in case of flow through gravel bed is insignificant, but porosity of filter media has got a significant effect on Reynold's Number.

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