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Ranking of alternatives in fuzzy environment using integral value

Kusum Deep, Associate Professor, Department of Mathematics, I.I.T.Roorkee, India, kusumfma@iitr.ernet.in
M.L.Kansal, Associate Professor, Department of Water Resources Development and Management, I.I.T.Roorkee, India, mlkgkfwt@iitr.ernet.in
K.P.Singh, Research Scholar, Department of Mathematics, I.I.T.Roorkee, India, kps_8@rediffmail.com

Abstract

Decision making in a multi-criterion fuzzy environment is a complex process. This requires the ranking of various possible alternatives with due consideration to the expert's opinion. Due to complexity of the problem, attempts have been made to suggest a more acceptable approach for ranking of various alternatives in fuzzy environment. In this study, the integral value approach with index of optimism of the decision maker has been used to rank the various alternatives. The proposed methodology has been illustrated and justified with the help of an example and the results are compared with the total utility or ordering value of each alternative based on maximizing and minimizing set approach reported in the literature.

Keywords: Fuzzy set; Fuzzy numbers; Integral value; Multicriteria decision making; Ranking alternatives.

1. Introduction

In multi-criteria decision making process, the ranking of alternatives on the basis of various expert's opinion has been an area of active research. The objective of such studies has been to estimate the preferential ranking of alternatives so that the decision maker takes the decision depending upon various constraints. The ranking of alternatives is carried out according to the highest degree of desirability with respect to various relevant criteria as well as on the basis of the expert's opinion. It is decided on the basis of estimated performance rating of each alternative on each of a given set of criteria. These performance values on the criteria's involved for each alternative are aggregated to form a preference rating and the alternative with the highest preference, indicating the best overall performance, is identified and ranked first. The alternative with next higher preference is ranked second and so on.

Most of the real-time decision making problems involve imprecise and incomplete data/ information, and thus the vagueness and fuzziness in specifying the performance of various alternatives under certain criteria. The classical MCDM methods cannot effectively handle problems with imprecise information.

These classical methods, involving both deterministic and random processes, tend to be less effective in conveying the imprecision and vagueness characteristics. Further, complexity increases with the involvement of various experts as the opinion of different experts varies due to subjectivity and competence of an individual. This has led to the development of fuzzy set theory (FST) by Zadeh [13], who proposed that the key elements in human thinking are not numbers but labels of fuzzy sets. FST is a powerful tool to handle imprecise data and fuzzy expressions that are more natural for humans than rigid mathematical rules and equations. In many fuzzy multi-criteria decision-making problems, the final scores of alternatives are represented in terms of fuzzy numbers. In order to rank the alternatives, the decision maker (DM) needs a method to establish a crisp total ordering from fuzzy numbers. In order to carry out the task of comparing fuzzy numbers, many authors [1,2, 3, 8, 10, 18, 22, 23, 24, 25, 26,27] have proposed the fuzzy ranking methods that yield a total ordered set of ranking. These methods can be used for a wide range of problems involving one fuzzy number attribute to many fuzzy number attributes, which may be trivial to complex in nature.

Generally, there are two approaches which are used for comparing the fuzzy numbers, which are :

(1) Define a ranking function, $f : P(\Re) \rightarrow \Re$ where $P(\Re)$ is the set of fuzzy numbers. Adamo [11], Yager [23], and Chang [28] have followed this approach.

(2) Obtain a fuzzy set of 'optimal alternatives', $\widetilde{D} = \{i \setminus \mu_D(i)\}$ where $\mu_D(i)$ is the degree to which i^{th} alternative will perform on desired optimal scale. It provides the possibility distribution of alternatives to be the best alternatives. Baas and Kwakernaak [24], Baldwin and Guild [8], Kerre [2], Watson et al. [25], Jain [18] and Chen [23] have followed this approach. Dubois and Prade [1] suggested the four grades of dominance and Campos and Gonzalez [14] suggested the average-value ranking method. Requena et al. [4, 5] and Cano et al. [7] suggested methods of automatic ranking of fuzzy numbers using artificial neural networks (ANN).

Chen[23] has used the maximizing and minimizing set approach for ranking the fuzzy numbers. It depends on $X_{\max}$ (the infimum of support set of the union of fuzzy weights which are being ranked), $X_{\min}$ (the suprimum of support set of the union of fuzzy weights which are being ranked) and K. Raj and Kumar [16] applied this approach for ranking the alternatives. However, Liou and Wang [26] have shown the drawback of maximizing and minimizing set approach. It was noticed that when $X_{\max}$, $X_{\min}$ and K are changed (due to addition or deletion of any alternative), the ranking of remaining alternatives may also change. Thus, the ranking order of alternatives is not consistent. They advocated the integral value approach for ranking the fuzzy numbers.

In this paper, we propose a method which overcomes this difficulty. The fuzzy weights of the alternatives are arrived at with the help of the fuzzy information supplied by several experts on alternatives under various criteria. The approach suggested by Buckley [6] has been used for the purpose which is based on pool first and pool last approach. Thereafter, the ranking of the alternatives is obtained using the concept of integral value approach. Further, an index of optimism is used to reflect the DM's optimistic attitude as suggested by Kim and Perk [11].

2. Description of the problem

Consider the problem of ranking m alternatives $\left(A_i\,; i = 1,2,.......,m\right)$ by a decision maker (DM). DM wishes to select from amongst *m* alternatives, with the help of information supplied by *n* experts $\left(E_j\,;\;\; j = 1,2.........n\right)$ about the alternatives for each of criteria $C_k\,;\;\; k = 1,2..........K$ and also the relative importance of each criteria with respect to some overall objective, which one best satisfy the criteria. The hierarchical structure which is used in this paper is shown in Fig.1.

Fuzzy Multi Criteria Decision Making (FMCDM) methods basically consist of two phases: Phase (I) is the aggregation of the performance ratings (or the degree of satisfactions) with respect to all criteria for each alternative, and Phase (II) is the ranking of the alternatives according to the overall aggregated performance ratings.

2.1. Phase (I)

2.1.1. Scale of preference

Let £= $l_1, l_2,......L$ be the preference information used by the experts. This scale is assumed to be finite, linearly ordered and $l_1 < l_2...... < L$. This £ can be an ordinal, an exact, a ratio, an interval scale or a combination of these scales. It may be easier for the experts to express their preferences in ordinal values (linguistic variables), especially, when there are more number of alternative and qualitative criteria, and when some of the criteria is vaguely understood or imprecisely defined. In this case the evaluation process may be very much subjective; however, it seems more appropriate to use ordinal scale than any other scales. For comparison amongst alternatives with respect to some of the criteria, experts may prefer ordinal values rather than numbers. The trapezoidal and triangular fuzzy numbers are the two main concepts used in this

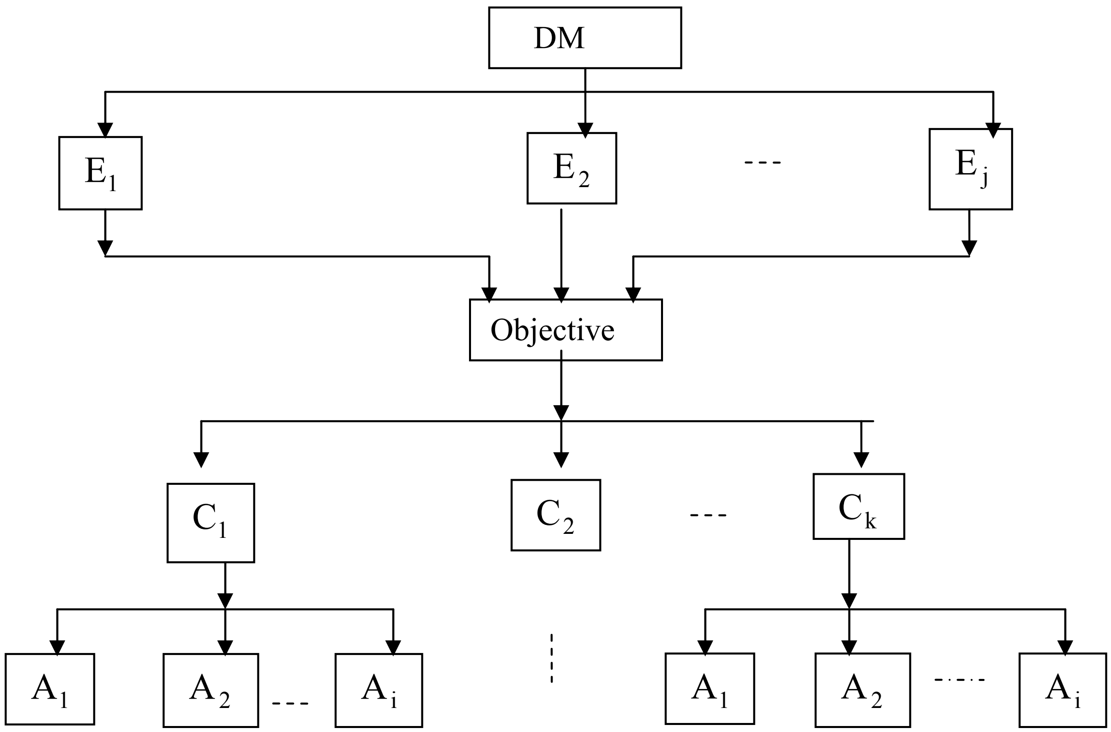

Fig.1: Hierarchical structure

study to assess the preference rating of linguistic variables, 'importance' and 'appropriateness'. The experts may employ an assumed weighting set W = {Very Low, Low, Medium, High, and Very High} to assess the relative importance of various criteria and use the linguistic rating set S = {Very Poor, Poor, Fair, Good, and Very Good} to evaluate the appropriateness of the alternatives versus various criteria. The membership functions of the linguistic values in the weighting set W and linguistic rating set S can be represented by the approximate reasoning of trapezoidal fuzzy numbers.

2.1.2. Fuzzy numbers

Let $\tilde{a}_i$ be a fuzzy number which is a fuzzy subset of $\Re$ (real numbers) and is considered in the form of $\tilde{a}_i = \{\alpha_i / \beta_i, \gamma_i / \delta_i\}, \quad i = 1,2......m$ (1)

where $\alpha < \beta < \gamma < \delta \in$ £, £ is the scale of preference information to be used by the experts.

2.1.3. Membership functions

Membership function of a fuzzy number $\tilde{a}_i$ is defined as:

$$f_{\tilde{a}_i}(x) = \begin{cases} 0, & x \le \alpha_i, \\ f^L{}_{\tilde{a}_i}(x), & \alpha_i \le x \le \beta_i, \\ 1, & \beta_i \le x \le \gamma_i, \\ f^R{}_{\tilde{a}_i}(x), & \gamma_i \le x \le \delta_i, \\ 0, & x \ge \delta_i, \end{cases} \tag{2}$$

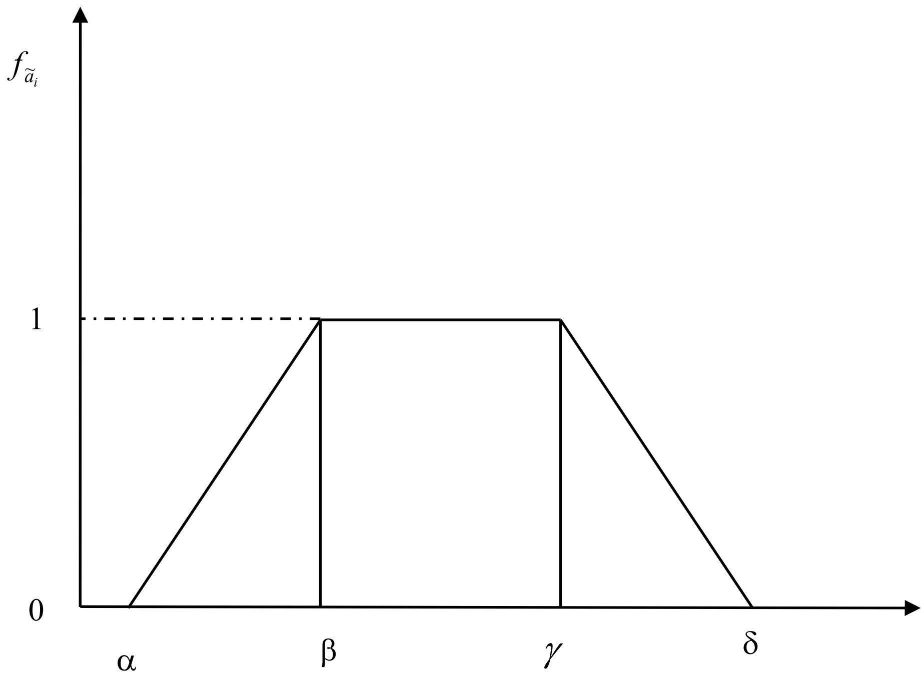

Fig.2: Graphical representation of trapezoidal fuzzy number

where $f^L{}_{\tilde{a}_i} : [\alpha_i, \beta_i] \to [0,1]$ and $f^R{}_{\tilde{a}_i} : [\gamma_i, \delta_i] \to [0,1]$. Since $f^L{}_{\tilde{a}_i} : [\alpha_i, \beta_i] \to [0,1]$ is continuous and strictly increasing, the inverse function of $f^L{}_{\tilde{a}_i}$ exists. Similarly, $f^R{}_{\tilde{a}_i} : [\gamma_i, \delta_i] \to [0,1]$ is continuous and strictly decreasing, the inverse function of $f^R{}_{\tilde{a}_i}$ also exists.

The inverse functions of $f^L{}_{\tilde{a}_i}$ and $f^R{}_{\tilde{a}_i}$ is denoted by $g^L{}_{\tilde{a}_i}$ and $g^R{}_{\tilde{a}_i}$ respectively. Since $f^L{}_{\tilde{a}_i} : [\alpha_i, \beta_i] \to [0,1]$ is continuous and strictly increasing, therefore, $g^L{}_{\tilde{a}_i} : [0,1] \to [\alpha_i, \beta_i]$ is also continuous and strictly increasing. Similarly, if $f^R{}_{\tilde{a}_i} : [\gamma_i, \delta_i] \to [0,1]$ is continuous and strictly decreasing, then $g^R{}_{\tilde{a}_i} : [0,1] \to [\gamma_i, \delta_i]$ is also continuous and strictly decreasing; $g^L{}_{\tilde{a}_i}$ and $g^R{}_{\tilde{a}_i}$ are continuous on a closed interval [0,1] and they are integrable on [0,1]. That is, $\int g^L{}_{\tilde{a}_i}(y)dy$ and $\int g^R{}_{\tilde{a}_i}(y)dy$ exists.

$$
f_{\tilde{a}_i} = \begin{cases} 0, & x \le \alpha_i, \\ (x-\alpha_i)/(\beta_i-\alpha_i), & \alpha_i \le x \le \beta_i, \\ 1, & \beta_i \le x \le \gamma_i, \\ (\delta_i - x)/(\delta_i-\gamma_i), & \gamma_i \le x \le \delta_i, \\ 0, & x \ge \delta_i, \end{cases} \tag{3}
$$

The trapezoidal membership function of an alternative $\tilde{a}_i$ is considered in the form of Eq. (3), which is shown in Fig.2. If $\beta_i = \gamma_i$ then it represents triangular fuzzy number and

$\alpha_i = \beta_i = \gamma_i = \delta_i$ represents a crisp number.

2.1.4. Rating assignment by the experts

Let the experts ($E_j; j = 1,2..........n$) assign rating in terms of fuzzy numbers or linguistic variable to the alternatives ($A_i; i = 1,2.........m$) for each of *K* criteria ($C_k; k = 1,2.......K$) and also to each criteria. Let

$$
\tilde{a}_{ij}^k = \left(\alpha^k{}_{ij} / \beta^k{}_{ij}, \gamma^k{}_{ij} / \delta^k{}_{ij}\right) \tag{4}
$$

be the fuzzy number assigned to alternative A_i by expert E_j for criteria C_k. This means that $\tilde{a}^k{}_{ij}$ measures how well A_i satisfies C_k for expert E_j. For each criterion k, the corresponding membership function can be represented as $f^j{}_{\tilde{a}_i}(x)$ (similar to Eq. (3)) and this data can be expressed in the matrix form as shown in Eq.(5).

$$
R_k = A_i \overset{\begin{matrix} E_1 & E_2 & & E_n \end{matrix}}{\begin{bmatrix} & & & \\ f^j{}_{\tilde{a}_i}(x) = \tilde{a}_{ij}^k & & & \\ & & & \end{bmatrix}}. \tag{5}
$$

Similarly, let

$$
\tilde{C}_{kj} = \left(\varepsilon_{kj} / \zeta_{kj}, \eta_{kj} / \theta_{kj}\right) \tag{6}
$$

be a fuzzy number given to criteria C_k by expert E_j. Thus, $\tilde{C}_{kj}$ indicates the importance of C_k for expert E_j with respect to an overall objective. The membership function of these fuzzy numbers can be represented as $f^j{}_{C_k}(x)$ and in matrix form as in Eq. (7)

$$
R = C_k \overset{\begin{matrix} E_1 & E_2 & & E_n \end{matrix}}{\begin{bmatrix} & & & \\ f^j{}_{C_k}(x) = \tilde{c}_{kj} & & & \\ & & & \end{bmatrix}}. \tag{7}
$$

2.1.5. Aggregation of performance rating

Given the data R_k and R, the DM computes the fuzzy weights ($\widetilde{w}_i ; i = 1,2.........m$) of the alternatives. The fuzzy weights for each of the alternatives can be derived at by pooling, averaging or aggregating across experts. This task can be performed in two ways; i.e., “pool-first" and "pool-last".

2.1.5.1. Pool-first approach

The pool first approach is as follows:
*Step1.*Find the averages of fuzzy numbers across all the experts first as

$$\tilde{P}_{ik} = (1/n) \otimes \left(\tilde{a}_{i1}^k \oplus \tilde{a}_{i2}^k \oplus ...... \oplus \tilde{a}_{in}^k\right) \text{And } \tilde{Q}_k = (1/n) \otimes \left(\tilde{C}_{k1} \oplus \tilde{C}_{k2} \oplus ....\tilde{C}_{kn}\right), \tag{8}$$

$\widetilde{P}_{ik}, \widetilde{Q}_k \in £$ where $\oplus$ and $\otimes$ are fuzzy addition and multiplication, respectively.
The fuzzy numbers shown in Eq. (8) are simply the row averages of matrices given in Eqs. (5) and (7) respectively, $\widetilde{P}_{ik}$ is the fuzzy ranking of A_i for criteria C_k and $\widetilde{Q}_k$ is the fuzzy ranking of C_k.

*Step2.*To determines the fuzzy weights ($\widetilde{w}_i$) of the alternatives, multiply $\widetilde{P}_{iK}$ and $\widetilde{Q}_K$, find the average over all criteria as

$$\tilde{w}_i = (1/KL) \otimes \left\{\left(\tilde{P}_{i1} \otimes \tilde{Q}_1\right) \oplus \left(\tilde{P}_{i2} \otimes \tilde{Q}_2\right) \oplus .... \oplus \left(\tilde{P}_{iK} \otimes \tilde{Q}_K\right)\right\}. \tag{9}$$

2.1.5.2 Pool-last approach

The pool first approach is as follows:
Step1 . Compute fuzzy weight $\widetilde{w}_{ij}$ for alternative A_i to each of the expert $\mathcal{E}_j$. $\widetilde{w}_{ij}$ is the fuzzy average over all the criteria and can be estimated as

$$\tilde{w}_{ij} = (1/KL) \otimes \left\{\left(\tilde{a}_{ij}^1 \otimes \tilde{c}_{1j}\right) \oplus \left(\tilde{a}_{ij}^2 \otimes \tilde{c}_{2j}\right) \oplus .... \oplus \left(\tilde{a}_{ij}^K \otimes \tilde{c}_{Kj}\right)\right\} \quad . \tag{10}$$

*Step2.*To determine final weight $(\widetilde{w}_i')$ of the alternatives, pooled fuzzy weight ($\widetilde{w}_{ij}$) across all the experts as

$$\tilde{w}_i' = (1/n) \otimes \left(\tilde{w}_{i1} \oplus \tilde{w}_{i2} \oplus .... \oplus \tilde{w}_{in}\right). \tag{11}$$

In Pool-First approach fuzzy weight $\widetilde{w}_i$ may be easily computed using standard fuzzy arithmetic as shown below. Let $\alpha_{ik}, \beta_{ik}, \gamma_{ik}$ and δ_{ik} be averages across experts of $\alpha^k{}_{ij}, \beta^k{}_{ij}, \gamma^k{}_{ij}$ and $\delta^k{}_{ij}$, respectively. Similarly let $\varepsilon_k, \zeta_k, \eta_k$ and δ_k be defined as the averages across experts of $\varepsilon_{kj}, \zeta_{kj}, \eta_{kj}$ and δ_{kj}, respectively. i.e.,

$$\begin{gathered}
\alpha_{ik} = \left(\sum \alpha^k{}_{ij}\right)/n, \quad j = 1,2..........n, \quad \beta_{ik} = \left(\sum \beta^k{}_{ij}\right)/n, \quad j = 1,2......n \\
\gamma_{ik} = \left(\sum \gamma^k{}_{ij}\right)/n, \quad j = 1,2.......n, \quad \delta_{ik} = \left(\sum \delta^k{}_{ij}\right)/n, \quad j = 1,2........n. \\
\varepsilon_k = \left(\sum \varepsilon_{kj}\right)/n, \quad j = 1,2.........n \quad \zeta_k = \left(\sum \zeta_{kj}\right)/n, \quad j - 1,2.........n, \\
\gamma_k = \left(\sum \gamma_{kj}\right)/n, \quad j = 1,2.........n, \quad \theta_k = \left(\sum \theta_{kj}\right)/n, \quad j = 1,2........n.
\end{gathered} \tag{12}$$

Let the fuzzy weight be described as :

$$\tilde{w}_i = \left(\alpha_i\left[L_{i1}, L_{i2}\right] / \beta_i, \gamma_i / \delta_i\left[U_{i1}, U_{i2}\right]\right). \tag{13}$$

Where,

$$\alpha_i = \left(\sum \alpha_{ik}\varepsilon_k\right) / KL,$$
$$\beta_i = \left(\sum \beta_{ik}\zeta_k\right) / KL,$$
$$\gamma_i = \left(\sum \gamma_{ik}\eta_k\right) / KL,$$
$$\delta_i = \left(\sum \delta_{ik}\theta_k\right) / KL,$$
$$L_{i1} = \left\{\sum (\beta_{ik} - \alpha_{ik})(\zeta_k - \varepsilon_k)\right\} / KL,$$
$$L_{i2} = \left[\sum \left\{\alpha_{ik}(\zeta_k - \varepsilon_k) + \varepsilon_k(\beta_{ik} - \alpha_{ik})\right\}\right] / KL,$$
$$U_{i1} = \left\{\sum (\delta_{ik} - \gamma_{ik})(\theta_k - \eta_k)\right\} / KL,$$
$$U_{i2} = -\left[\sum \left\{\delta_{ik}(\theta_k - \eta_k) + \theta_k(\delta_{ik} - \gamma_{ik})\right\}\right] / KL.$$

The Graph of the membership function of $\tilde{w}_i$ is: zero to the left of α_i; $L_{i1}y^2 + L_{i2}y + \alpha_i = x$ on $\left[\alpha_i, \beta_i\right]$; horizontal line (y=1) between $\left[\beta_i, \gamma_i\right]$; $U_{i1}y^2 + U_{i2} + \delta_i$ on $\left[\gamma_i, \delta_i\right]$ and zero to the right of δ_i (x-axis is horizontal line and y-axis vertical). Theorems related to these equations, the proofs and properties are well described in Dubois and Prade[1] and Buckley[6]. Membership function of fuzzy weight $\tilde{w}_i$ is described as in Eq.(14), which is drum like shape.

$$f_{\tilde{w}_i}(x) = \begin{cases} 0, & x \le \alpha_{i'} \\ -L_{i2}/2L_{i1} + \left\{\left(L_{i2}/2L_{i1}\right)^2 + \left(x - \alpha_i\right)/L_{i1}\right\}^{1/2}, & \alpha_i \le x \le L_{i1}y^2 + L_{i2}y + \alpha_i, \\ 1, & L_{i1}y^2 + L_{i2}y + \alpha_i \le x \le U_{i1}y^2 + U_{i2}y + \delta_i, \\ -U_{i2}/2U_{i1} + \left\{\left(U_{i2}/2U_{i1}\right)^2 + \left(x - \delta_i\right)/U_{i1}\right\}^{1/2}, & U_{i1}y^2 + U_{i2}y + \delta_i \le x \le \delta_i, \\ 0, & x \ge \delta_i \end{cases} \tag{14}$$

from Eq.(2 and 14) $f^L_{\tilde{w}_i} = -L_{i2}/2L_{i1} + \left\{\left(L_{i2}/2L_{i1}\right)^2 + \left(x - \alpha_i\right)/L_{i1}\right\}^{1/2}$, and

$$f^R_{\tilde{w}_i} = -U_{i2}/2U_{i1} + \left\{\left(U_{i2}/2U_{i1}\right)^2 + \left(x - \delta_i\right)/U_{i1}\right\}^{1/2}. \tag{15}$$

Above discussion can be easily extend for pool-last approach. Here our discussion is for only pool-first approach.

2.2. Phase (II)

2.2.1. Integral value approach for ranking of alternatives

Integral value approach has been used, on the basis of aggregated fuzzy weights, to calculate the final ranking order of the alternatives. This method, which is independent of the type of membership function used and the normality of the function, can rank more than two fuzzy numbers simultaneously. It is relatively simple in computation. Further, an index of optimism has been used to reflect the DM's optimistic attitude.

Let $g^L{}_{\tilde{w}_i}$ and $g^R{}_{\tilde{w}_i}$ are inverse functions of $f^L{}_{\tilde{w}_i}$ and $f^R{}_{\tilde{w}_i}$ (Eq. 15), respectively.

$$g^L{}_{\tilde{w}_i} = L_{i1}y^2 + L_{i2}y + \alpha_i \text{, and } g^L{}_{\tilde{w}_i} = U_{i1}y^2 + U_{i2}y + \delta_i . \tag{16}$$

Left integral value of $\tilde{w}_i$ is defined as

$$\begin{aligned} I_L(\tilde{w}_i) &= \int_0^1 g^L{}_{\tilde{w}_i}(y)\,dy \\ &= L_{i1}/3 + L_{i2}/2 + \alpha_i \end{aligned} \tag{17}$$

and Right integral value of $\tilde{w}_i$ is defined as

$$\begin{aligned} I_R(\tilde{w}_i) &= \int_0^1 g^R{}_{\tilde{w}_i}(y)\,dy \\ &= U_{i1}/3 + U_{i2}/2 + \delta_i . \end{aligned} \tag{18}$$

Total integral value with index of optimism, μ [0, 1], is defined as

$$\begin{aligned} I^\mu{}_T(\tilde{w}_i) &= \mu I_R(\tilde{w}_i) + (1-\mu) I_L(\tilde{w}_i). \\ &= \mu\left[U_{i1}/3 + U_{i2}/2 + \delta_i\right] + (1-\mu)\left[L_{i1}/3 + L_{i2}/2 + \alpha_i\right]. \end{aligned} \tag{19}$$

The index of optimism (μ) represents degree of optimism for a decision maker. A larger μ indicates a higher degree of optimism. More Specifically, when μ=0, total integral value $I^0{}_T(\tilde{w}_i)$ which represents a pessimistic DM's viewpoint is equal to the left integral value of $\tilde{w}_i$ i.e $I_L(\tilde{w}_i)$. Conversely, for an optimistic decision maker, i.e. μ=1, total integral value $I^1{}_T(\tilde{w}_i)$ is equal to $I_R(\tilde{w}_i)$. For a moderate decision maker, with μ=0.5, total integral value becomes $I^{0.5}{}_T = 1/2\left[I_R(\tilde{w}_i) + I_L(\tilde{w}_i)\right]$.

The total integral value function $I^\mu{}_T(\tilde{w}_i)$ has the following properties:

(1) if $I^\mu{}_T(\tilde{w}_i) < I^\mu{}_T(\tilde{w}_j)$ then $\tilde{w}_i < \tilde{w}_j$,

(2) if $I^\mu{}_T(\tilde{w}_i) = I^\mu{}_T(\tilde{w}_j)$ then $\tilde{w}_i = \tilde{w}_j$,

(3) if $I^\mu{}_T(\tilde{w}_i) > I^\mu{}_T(\tilde{w}_j)$ then $\tilde{w}_i > \tilde{w}_j$.

The proposed approach has the following steps:

Step1. Form a committee of experts, identify the selection attributes, (cost or benefit) and list all possible alternatives.
Step2. Collect each expert's opinion for each alternative with respect to a subjective attribute and establish a decision matrix for each expert.
Step3. Transform the fuzzy data (linguistic expressions or fuzzy assessments) into standardized positive trapezoidal fuzzy numbers attribute.
Step4. Assign relative importance of criteria for each expert.

Step5. Under each subjective criteria, aggregate all expert's fuzzy opinions for each alternative using Eqs(8)–(14). This step gives us fuzzy weight of alternatives.
Step6. Order or rank the alternatives according to the total integral values and select the alternative with the maximum total integral value as the best alternative.

3. Illustrative Example

A decision maker wishes to rank three alternatives $(A_i; i = 1,2,3,)$ across the two criteria $(C_K; K = 1,2)$ using the information supplied by four experts $(E_j; j = 1,2,3,4)$. The fuzzy numbers used by the experts are $\alpha, \beta, \gamma, \delta \in$ £ (0, 1, 2………, 20). For the qualitative (linguistic) evaluation, these experts may use standard fuzzy numbers, suggested by DM, or different fuzzy numbers. In this example, let us suppose that, the experts have used different fuzzy numbers. Let us also assume that the DM uses pool-first approach before arriving at final fuzzy weights.

3.1. Assignment and Aggregation of preference rating

Fuzzy weights of criterion by the experts are given in Table 1, which shows the importance of criteria in overall objective. The preference of the experts for the alternatives corresponding to criteria C_1 and C_2 are given in Table 2 and Table 3, respectively.

Table 1: Ranking of criteria by experts

	E_1	E_2	E_3	E_4
C_1	(12/14,16/18)	(8/10,12/14)	(10/12,14/16)	(10/12,14/16)
C_2	(6/8,10/12)	(12/14,14/14)	(0/1,2/2)	(6/8,10/12)

Table 2: Ranking of alternatives for criteria C_1 by experts

C_1	E_1	E_2	E_3	E_4
A_1	(6/7,7/8)	(4/4,5/5)	(0/1,2/2)	(3/4,5/6)
A_2	(5/7,7/8)	(3/4,5/6)	(0/1,2/3)	(2/4,5/7)
A_3	(3/5,5/5)	(6/7,7/8)	(4/5,5/6)	(5/6,6/7)

Table 3: Ranking of alternatives for criteria C_2 by experts

C_2	E_1	E_2	E_3	E_4
A_1	(6/7,7/8)	(6/7,7/8)	(8/8,8/8)	(6/7,8/9)
A_2	(5/7,7/8)	(5/7,7/9)	(7/8,8/9)	(5/7,8/9)
A_3	(3/4,4/5)	(5/6,6/7)	(4/5,6/6)	(4/6,7/7)

Fuzzy weights of alternatives, after applying pool-first approach, are-

$$f_{\tilde{w}_1}(x)=\begin{cases}-.7469/2(.0703)+\left\{(.7469/2(.0703))^2+(x-1.7875)/.0703\right\}^{1/2}, \\ \qquad .7875\le x\le .0703y^2+.7469y+1.7875, \\ 1\ , \qquad .0703y^2+.7469y+1.7875\le x\le .0437y^2-.8566y+4.1625, \\ .8562/2(.0437)+\left\{(-.8562/2(.0437))^2+(x-4.1625)/.0437\right\}^{1/2}, \\ \qquad .0437y^2-.8566y+4.1625\le x\le 4.1625,\end{cases}$$

$$f_{\tilde{w}_2}(x)=\begin{cases}-1.0031/2(.1516)+\left\{(1.0031/2(.1516))^2+(x-1.45)/.1516\right\}^{1/2}, \\ \qquad 1.45\le x\le .1516y^2+1.0031y+1.45, \\ 1\ , \qquad .1516y^2+1.0031y+1.45\le x\le .0938y^2-1.3313y+4.5875, \\ 1.3313/2(.0938)+\left\{(-1.3313/2(.0938))^2+(x-4.5875)/.0938\right\}^{1/2}, \\ \qquad .0938y^2-1.3313y+4.5875\le x\le 4.5875,\end{cases}$$

$$f_{\tilde{w}_3}(x)=\begin{cases}-.9/2(.1172)+\left\{(.9/2(.1172))^2+(x-1.7250)/.1172\right\}^{1/2}, \\ \qquad 1.7250\le x\le .1172y^2+.9y+1.7250, \\ 1\ , \qquad .1172y^2+.9y+1.7250\le x\le .05y^2-.9063y+4.1625, \\ .9063/2(.05)+\left\{(-.9063/2(.05))^2+(x-4.1625)/.05\right\}^{1/2}, \\ \qquad .05y^2-.9063y+4.1625\le x\le 4.1625,\end{cases}$$

3.2. Ranking of alternatives

Ranking order of alternatives A_1, A_2 and A_3 with different degree of optimism (μ) of DM as suggested earlier have been calculated and results are shown in Table 4. Results, in Table 4, show that as the value of μ is changed, ranking order of alternatives also changes. For pessimistic (μ =0) DM ranking order is $A_3 > A_1 > A_2$, while for optimistic (μ =1) DM ranking order is $A_2 > A_1 > A_3$.

Table 4: Result by proposed method with different degree of optimism of decision maker

μ	0.0		0.25		0.5		0.75		1.0	
Ranking	Alt	$I^{\mu}{}_T(\tilde{w}_i)$	Alt	$I^{\mu}{}_T(\tilde{w}_i)$	Alt	$I^{\mu}{}_T(\tilde{w}_i)$	Alt	$I^{\mu}{}_T(\tilde{w}_i)$	Alt	$I^{\mu}{}_T(\tilde{w}_i)$
1	A_3	1.1070	A_3	1.2960	A_2	1.4888	A_2	1.7327	A_2	1.9766
2	A_1	1.0922	A_1	1.2878	A_3	1.4850	A_1	1.6789	A_1	1.8745
3	A_2	1.0010	A_2	1.2449	A_1	1.4833	A_3	1.6740	A_3	1.8630

In order to compute the results with maximization and minimization set approach (Raj and Kumar [16]), the ranking of alternative was carried out by this approach and is shown in Table 5. This shows ranking order is $A_3 > A_2 > A_1$.

Table 5: Result by maximizing and minimizing set approach

Ranking	1	2	3
Alternative	A_3	A_2	A_1
Ranking Value	0.4907	0.4875	0.4863

After getting the ranking order of three alternatives, suppose DM introduces another alternative A_4 for ranking with above alternatives whose experts opinion for criteria C_1 and C_2 are given in Table 6 and Table 7, respectively. Membership function of alternative A_4, by pool-first approach, is

$$f_{\tilde{w}_4}(x)=\begin{cases} -1.74006/2(.1906)+\left\{\left(1.74006/2(.1906)\right)^2+(x-3.8)/.1906\right\}^{1/2}, \\ \qquad 3.8\le x\le .1906y^2+1.74006y+3.8, \\ 1, \qquad .1906y^2+1.74006y+3.8\le x\le .1250y^2-2.175y+9.3, \\ 2.175/2(.1250)+\left\{\left(-2.175/2(.1250)\right)^2+(x-9.3)/.1250\right\}^{1/2}, \\ \qquad .1250y^2-2.175y+9.3\le x\le 9.3, \end{cases}$$

Table 6: Ranking of alternatives for criteria C_1 by experts

C_1	E_1	E_2	E_3	E_4
A_4	(8/9,10/10)	(14/16,16/18)	(10/10,12/14)	(6/12,14/16)

Table 7: Ranking of alternatives for criteria C_2 by experts

C_2	E_1	E_2	E_3	E_4
A_4	(8/10,10/12)	(10/12,12/14)	(10/10,12/14)	(10/12,14/16)

Now, ranking order of all four alternatives, again, has been found with both the techniques. Ranking order of all four alternatives with different degree of optimism for DM, by integral value approach, is shown in Table 8. For pessimistic (μ=0) DM ranking order of alternatives is $A_4 > A_3 > A_1 > A_2$, while optimistic (μ=1) DM ranking order of alternatives is $A_4 > A_2 > A_1 > A_3$. Ranking order of alternatives by maximizing and minimizing set approach is shown in Table 9, which is $A_4 > A_2 > A_3 > A_1$.

Table 8: Result by proposed method with different degree of optimism of decision maker

μ	0.0		0.25		0.5		0.75		1.0	
Ranking	Alt	$I^{\mu}{}_T(\tilde{w}_i)$	Alt	$I^{\mu}{}_T(\tilde{w}_i)$	Alt	$I^{\mu}{}_T(\tilde{w}_i)$	Alt	$I^{\mu}{}_T(\tilde{w}_i)$	Alt	$I^{\mu}{}_T(\tilde{w}_i)$
1	A_4	2.3669	A_4	2.8070	A_4	3.2470	A_4	3.6870	A_4	4.1271
2	A_3	1.1070	A_3	1.2960	A_2	1.4888	A_2	1.7327	A_2	1.9766
3	A_1	1.0922	A_1	1.2878	A_3	1.4850	A_1	1.6789	A_1	1.8745
4	A_2	1.0010	A_2	1.2449	A_1	1.4833	A_3	1.6740	A_3	1.8630

Table 9: Result by maximizing and minimizing set approach

Ranking	1	2	3	4
Alternative	A_4	A_2	A_3	A_1
Ranking Value	0.6094	0.2483	0.2409	0.2357

3.3. Results and Discussion

On the basis of these results, it is observed that when a new alternative A_4 is introduced then the results are changed while using maximizing and minimizing set approach. When only three alternatives, A_1, A_2 and A_3 are ranked then the ranking order is $A_3 > A_2 > A_1$, and when additional alternative A_4 is introduced then the ranking order is $A_4 > A_2 > A_3 > A_1$. In first ranking results $A_3 > A_2$, and when A_4 is introduced in ranking process then $A_2 > A_3$, i.e., in ranking process an alternative value is affected by another alternative.

In this study, misleading results by maximizing and minimizing set approach has been avoided using integral value approach. Ranking order of first three alternatives is not affected, for an optimism level of DM's, by introducing another alternative A_4. DM's degree of optimism is also incorporated in integral value approach; therefore, he gets ranking order of alternatives for different degree of optimism. Example shows that ranking order of alternatives is also influenced by DM's optimism level. For $\mu = 0$ (pessimistic DM) ranking order of alternatives is $A_4 > A_3 > A_1 > A_2$ and for $\mu = 1$ (optimistic DM) ranking order is $A_4 > A_2 > A_1 > A_3$.

4. Conclusions

The proposed procedure is intuitive, computationally simple and easy to implement in ranking of alternative, which has lot of potential for making policy decisions in a large-scale, real-life and complex problems. Both qualitative and quantitative aspects are analyzing by this method, employing expert's opinion (preference structure) using fuzzy numbers and linguistic variables. Numerical results show the efficiency and reliability of integral value approach over maximizing and minimizing set approach in multicriteria decision making problems.

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