

Boundaries of thermal stability of a vibrationally nonequilibrium flowing gas considering constant energy pumping power

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Abstract

The boundaries of the existence of a vibrationally nonequilibrium gas was theoretically studied. The gas was flowing in a circular tube with fixed surface temperature and the nonequilibrium state was maintained by pumping energy into the gas vibrational modes by an external source of power I . Results showed that the gas vibrational energy ϵ increases by increasing I to a certain maximum value I_m above which ϵ took a fixed maximum value ϵ_m and the gas got heated. The variation of ϵ_m with Reynolds number, the density and type of the gas, the length and diameter of the tube was determined.

Introduction

A vibrationally nonequilibrium gas is a molecular gas with excess of vibrational energy, which is supplied by an external source of energy. The release of this energy into gas translational modes greatly affects stability of the gas nonequilibrium state. This was theoretically shown in [1-3] for a flat layer of a fixed (no flow) gas. In [1] the stability with respect to acoustic perturbations of a nonuniform vibrationally nonequilibrium gas with constant external energy pumping power was studied. In [2] thermal stability of a vibrationally nonequilibrium gas was studied in frame of thermal explosion theory. Thermal explosion constitutes a sharp increase in temperature in an enclosed volume when heat released has no time to dissipate into the surrounding environment. Fast release of excess vibrational energy into gas translational modes leads to sudden increase in gas temperature and the nonequilibrium gas changes to equilibrium. The change takes place at certain energy pumping power, which was determined in [2]. In [3] the conditions under which convective, thermal, and acoustic instabilities arise are compared, and the regions of thermodynamic parameters at which a specific type of instability is dominant are determined. In this paper thermal stability of a vibrationally nonequilibrium gas considering constant external energy pumping power flowing in a circular tube is theoretically studied.

Theory

A state of nonequilibrium vibrational energy can be maintained by supplying energy into vibrational modes of the gas by an external source (for example laser radiation as a source of constant energy power) and removing heat from the gas into its surrounding. In gas harmonic oscillators limit, hydrodynamically fully-developed laminar flow in a circular tube of a vibrationally nonequilibrium incompressible gas considering constant energy pumping power is described as follows:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - \frac{\rho}{\lambda_T} C_T U \frac{\partial T}{\partial x} + \frac{\rho}{\lambda_T} \frac{\frac{k\theta}{m} \left(\exp\left(\frac{\theta}{T_v}\right) - 1 \right)^{-1} - \frac{k\theta}{m} \left(\exp\left(\frac{\theta}{T}\right) - 1 \right)^{-1}}{A \exp\left(\frac{B}{T^{1/3}}\right)} = 0, \quad (1)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_v}{\partial r} \right) - \frac{\rho}{\lambda_v} C_v U \frac{\partial T_v}{\partial x} + \frac{\rho}{\lambda_v} I - \frac{\rho}{\lambda_v} \frac{\frac{k\theta}{m} \left(\exp\left(\frac{\theta}{T_v}\right) - 1 \right)^{-1} - \frac{k\theta}{m} \left(\exp\left(\frac{\theta}{T}\right) - 1 \right)^{-1}}{A \exp\left(\frac{B}{T^{1/3}}\right)} = 0, \quad (2)$$

$$\int_S \lambda_v \frac{\partial T_v}{\partial r} ds + \int_S \lambda_T \frac{\partial T}{\partial r} ds + \int_{Out} \rho U C_T T dA_C - \int_{in} \rho U C_T T dA_C = \rho I L A_C, \quad (3)$$

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dU}{dr} \right) - \frac{1}{\mu} \frac{dP}{dx} = 0, \quad (4)$$

where T is the gas translational temperature, T_v is the gas vibrational temperature, λ_T is the gas coefficient for translational heat conduction, λ_v is the gas coefficient for vibrational heat conduction, C_T is the gas specific heat capacity of gas translational and rotational modes, C_v is the gas specific heat

capacity of gas vibrational modes, θ is the characteristic vibrational temperature of the gas, A and B are parameters that characterize the gas, I is the energy pumping power (constant), U is the gas velocity, μ is the gas viscosity coefficient, ρ is the gas density (constant), k is Boltzmann constant, P is the gas pressure, L is the tube length, and A_c is its cross section ($A_c = \pi r_0^2$, where r_0 is the tube radius). The tube axis is along x-direction and gas flows in the x-direction. Gas velocity U is a function of r and $\frac{\partial U}{\partial x} = 0$.

The pressure gradient is independent of r . The coefficients μ , λ_T , λ_V , in addition to C_T and C_V are considered constant.

The balance of gas translational energy is expressed in Eq.(1). The third term on the left of Eq.(1) is the source of energy for gas translational modes. It is the released vibrational energy into translational modes according to Landau-Teller relation of the rate of vibrational energy change of simple harmonic oscillators [4]. The balance of gas vibrational energy is given in Eq. (2). The third term on the left of Eq.(2) is the supplied energy by an external source into gas vibrational modes. The fourth term on the left of Eq.(2) is the vibrational energy loss into translational modes by vibrational energy relaxation. The first term on the left of Eq.(3) is the flux of heat into the tube surface by vibrational heat conduction. The second term on the left of Eq.(3) is the flux of heat into the tube surface by translational heat conduction. The third term and the fourth term on the left of Eq.(3) is the net heat energy carried out of the gas through outlet cross section A_c by the flow. The term on the right of Eq.(3) is the flux of energy into the gas from an external source. Distribution of gas velocity along the tube diameter is explained by Eq.(4). The conditions on the boundaries are: $U=0$ (no-slip), $T= T_S$ (a fixed temperature T_S on the tube surface), and $T_V = T_S$.

The flow is characterized by Reynolds number $Re = \frac{2 \rho U_m r_0}{\mu}$, where U_m is the mean flow velocity

[5]. Other non-dimensional quantity $\alpha = \frac{\rho \mathcal{E}(T_S) r_0^2}{\lambda_V(T_S) \tau(\rho, T_S) T_S}$, which contains many parameters affecting gas stability, can be deduced from the following non-dimensional form of Eq.(2):

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T_V^*}{\partial r^*} \right) + I^* - \alpha \frac{(\mathcal{E}^* - \mathcal{E}_0^*)}{\tau^*} = 0, \quad (5)$$

and the following non-dimensional variables are used: $r^* = \frac{r}{r_0}$, $T_V^* = \frac{T_V}{T_S}$, $I^* = \frac{\rho I r_0^2}{\lambda_V(T_S) T_S}$,

$\tau^* = \frac{\tau(\rho, T)}{\tau(\rho, T_S)}$, $\mathcal{E}^* = \frac{\mathcal{E}(T_V)}{\mathcal{E}(T_S)}$, $\mathcal{E}_0^* = \frac{\mathcal{E}(T)}{\mathcal{E}(T_S)}$, where $\tau(\rho, T)$ is the gas vibrational energy relaxation time at the tube surface temperature ($\tau(\rho, T) = A \exp(B/T^{1/3})$), $\mathcal{E}(T_V)$ is the gas vibrational energy at the gas vibrational temperature T_V ($\mathcal{E}(T_V) = \frac{k\theta}{m} \left(\exp\left(\frac{\theta}{T_V}\right) - 1 \right)^{-1}$), $\mathcal{E}(T_S)$ is the gas vibrational energy at the tube surface temperature T_S , $\mathcal{E}(T)$ is the gas vibrational energy at the gas temperature T , and I^* is the non-dimensional energy pumping power. In addition to Re , δ , and I^* the non-dimensional quantity $\gamma = \frac{L}{r_0}$ is considered.

Results and discussion

A vibrationally equilibrium gas, i.e. its translational T and vibrational T_V temperatures are the same, inters tube of length L and radius r_0 with fixed surface temperature T_S ($T = T_V = T_S$). An external source of energy provides energy into gas vibrational modes in the bulk of the tube with constant pumping power I , such as laser radiation. The gas vibrational modes gain energy and due to fast redistribution of the energy among them a vibrational temperature $T_V > T$ is defined. The vibrationally nonequilibrium gas losses its excess energy by vibrational energy relaxation into its translational modes. The gas temperature rises above the tube surface temperature and heat transfer takes place from the gas into the tube surface, consequently a distribution of gas temperature forms along the tube diameter.

Results of numerical integration of Eq.(1), Eq.(2), and Eq.(4) show that increase of the energy pumping power I results in rise of the maximum gas temperature T_{max} and the gas vibrational energy $\mathcal{E} = \mathcal{E}(T_V)$ as illustrated in figure 1 for $Re = 200$, $\gamma = 20$, and $\alpha = 0.1 \times 10^{-4}$.

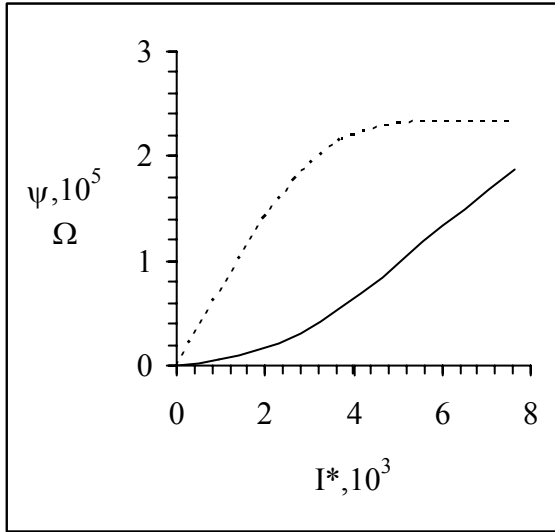


Figure 1. The variation of the non-dimensional gas vibrational energy ψ (the dotted curve) and the non-dimensional maximum gas temperature Ω (the solid curve) with the non-dimensional energy pumping power I^* for $Re = 200$, $\gamma = 20$, and $\alpha = 0.1 \times 10^{-4}$.

In figure1 on the vertical axis are the non-dimensional maximum gas temperature

$$\Omega = \frac{T_{\max} - T_s}{T_s} \quad (\text{the solid curve})$$

and the non-dimensional vibrational energy

$$\psi = \frac{\int \rho \mathcal{E}(T_v) dv}{\rho \mathcal{E}(T_s) L A_C} \quad (\text{the dotted curve}),$$

where the value of the integral is the gas vibrational energy and v is the gas volume.

On the horizontal axis is the non-dimensional energy pumping power I^* . ψ

increases with increasing I^* up to a certain value $I^* = I_m^*$ then it takes a fixed value

$$\psi = \psi_m.$$

The non-dimensional maximum gas temperature Ω increases slowly with increasing energy pumping power up to I_m^* after that it increases rapidly. In figure1 $I_m^* = 4.4 \times 10^3$, $\psi_m = 2.27 \times 10^5$, and $\Omega = 0.76$. For $I^* > I_m^*$ the gas gets heated and gas vibrational energy does not increase further. I_m^* is the maximum energy pumping power to maintain a vibrationally nonequilibrium state, i.e. $T_v \gg T$, with no excessive heating of the gas and a maximum gas vibrational energy. This can be explained by the accelerated loss of energy of the gas vibrational modes. The gas vibrational modes lose energy by the process of vibrational energy relaxation, which is the forth term on the left side of Eq.(2). The vibrational relaxation time of the gas, $\tau(\rho, T) = A \exp(B/T^{1/3})$, depends exponentially on gas temperature. As energy pumping power increases the gas temperature increases so $\tau(\rho, T)$ shortens and more vibrational energy goes into the translational modes of the gas. In consequence the gas temperature rises further and this cycle continues leading to fast release of the gas vibrational energy and rapid growth of gas temperature. This limits the gas vibrational energy growth with energy pumping power rise.

The release of the gas vibrational energy into the bulk of the gas is shown in figure 2 in addition to the various processes of losing energy for $Re = 200$, $\gamma = 20$, and $\alpha = 0.1 \times 10^{-4}$. The solid curve, $RT =$

$$\frac{\int \rho \frac{(\mathcal{E}(T_v) - \mathcal{E}(T))}{\tau} dv}{\rho I L A_C},$$

is the fraction of the input energy into the gas goes out of the gas vibrational

modes into the bulk of the gas by the process of vibrational energy relaxation. Its contribution in decreasing gas vibrational energy increases by increasing the energy pumping power of the external

$$VC = \frac{\int \lambda_v \frac{\partial T_v}{\partial r} ds}{\rho I L A_C},$$

the dotted curve, is the fraction of the input energy into the gas

carried out into the tube surface by the process of vibrational heat conduction, assuming the relaxation of gas vibrational energy on the tube surface is fast (the first term on the left of Eq.(3)). At the lower values of energy pumping power it is the dominant process to carry the vibrational energy out of the gas. As the pumping energy power increases its contribution decreases and the contribution of gas flow in carrying heat energy out (the forth and third terms on the left of Eq.(3)) increases.

$$OF = \frac{\int_{\text{Out}} \rho U C_T T dA_C - \int_{\text{in}} \rho U C_T T dA_C}{\rho I L A_C},$$

the bold-dotted curve, is the fraction of the input energy into

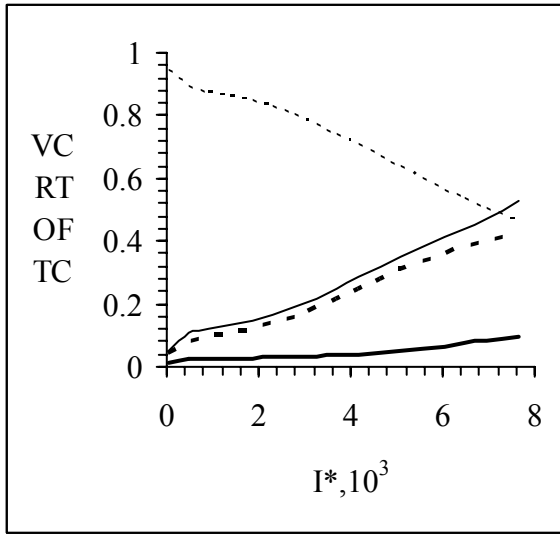


Figure 2. The variation of the fraction of the input energy carried out by vibrational heat conduction VC (the dotted curve), the fraction carried out by vibrational energy relaxation RT (the solid curve), the fraction carried out by gas flow OF (the bold-dotted curve), and the fraction carried out by translational heat conduction TC (the bold-solid curve) with the non-dimensional energy pumping power I^* for $Re=200$, $\gamma = 20$, and $\alpha = 0.1 \times 10^{-4}$.

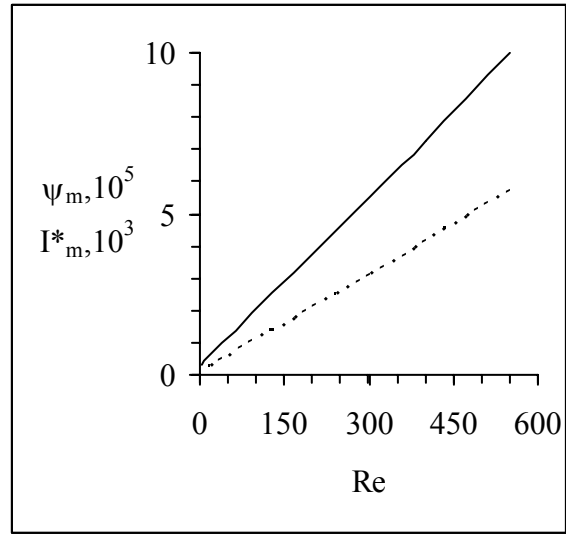


Figure 3. The variation of the non-dimensional maximum vibrational energy ψ_m (the dotted curve) and the non-dimensional maximum energy pumping power I_m (the solid curve) with Reynolds number Re .

the gas carried out by the flow of the gas. $TC = \frac{\int \lambda_T \frac{\partial T}{\partial r} ds}{\rho I L A_c}$, the bold-solid curve, is the fraction of the input energy into the gas carried out into the tube surface by the process of translational heat conduction (the second term on the left of Eq.(3)). Its contribution in carrying the heat out of the gas is the least.

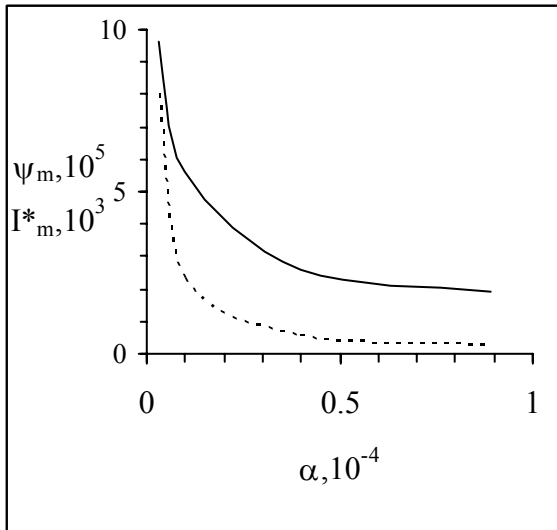


Figure 4. The variation of the non-dimensional maximum vibrational energy ψ_m (the dotted curve) and the non-dimensional maximum energy pumping power I_m (the solid curve) with α .

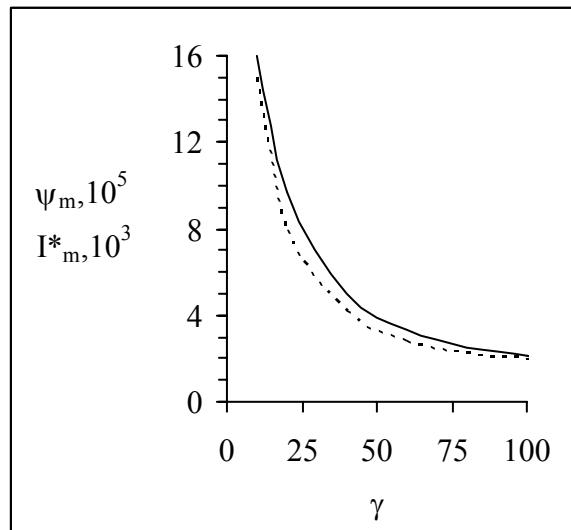


Figure 5. The variation of the non-dimensional maximum vibrational energy ψ_m (the dotted curve) and the non-dimensional maximum energy pumping power I_m (the solid curve) with γ .

The variation of the non-dimensional maximum vibrational energy ψ_m and the non-dimensional maximum energy pumping power I_m^* with Reynolds number Re is depicted in Figure 3 for $\gamma = 20$, and $\alpha = 0.1 \times 10^{-4}$. ψ_m (the dotted curve) and I_m^* (the solid curve) linearly increase with increasing Re . ψ_m and I_m^* increase by decreasing α and γ as shown in figure 4 and figure 5.

In figure 4 the variation of ψ_m (the dotted curve) and I_m^* (the solid curve) with α is shown for $Re=200$ and $\gamma = 20$. ψ_m is significant for $\alpha < 0.2 \times 10^{-4}$. The variation of ψ_m (the dotted curve) and I_m^* (the solid curve) with γ is illustrated in figure 5 for $Re = 200$ and $\alpha = 0.03 \times 10^{-4}$. ψ_m is significant for $\gamma < 50$.

Conclusion

The relationship between the maximum vibrational energy of a vibrationally nonequilibrium gas considering fixed external energy pumping power and the parameters of the relaxing flowing gas in a tube was determined.

The gas vibrational energy $\mathcal{E} = \mathcal{E}(T_v)$ increases with increasing the energy pumping power I up to $I = I_m$, where it takes a fixed value $\mathcal{E} = \mathcal{E}(T_v)_m$. I_m is the maximum energy pumping power for maintaining a vibrationally nonequilibrium state of the gas with no excessive heating of the gas and a maximum gas vibrational energy.

The loss of energy from the gas vibrational modes by vibrational energy relaxation into the gas translational modes increases with increasing the energy pumping power I . By increasing the energy pumping power the contribution of vibrational heat conduction in carrying the gas vibrational energy out of the gas vibrational modes into the tube surface decreases while the contribution of gas flow in carrying the heat out of the gas increases and dominates the contribution of the translational heat conduction in carrying the heat into the tube surface.

$$\int \rho \mathcal{E}(T_v) dv$$

In terms of the non-dimensional vibrational energy $\psi = \frac{v}{\rho \mathcal{E}(T_s) L A_C}$ and the non-dimensional

energy pumping power $I^* = \frac{\rho I r_0^2}{\lambda_v(T_s) T_s}$, the maximum non-dimensional vibrational energy is $\psi_m =$

2.27×10^5 and the maximum non-dimensional energy pumping power is $I_m^* = 4.4 \times 10^3$ for $Re = 200$, $\gamma = 20$, and $\alpha = 0.1 \times 10^{-4}$.

$\mathcal{E}(T_v)_m$ increases linearly with increasing Reynolds number Re . It decreases rapidly with increasing $\alpha = \frac{\rho \mathcal{E}(T_s) r_0^2}{\lambda_v(T_s) \tau(\rho, T_s) T_s}$ and $\gamma = \frac{L}{r_0}$. It is significant for $\alpha < 0.2 \times 10^{-4}$ when $Re=200$ and $\gamma = 20$ and for $\gamma < 50$ when $Re=200$ and $\alpha = 0.03 \times 10^{-4}$.

References

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